Hamming Error Correction Example

Suppose an 8-bit word stored in memory is 1100 0010. Using the Hamming Algorithm, determine what check bits would be stored in memory with data word?.

step①: Determine the number of check bits needed:

$$2^k - 1 \geq M + k$$

k = # of check bits
M = length of word

~~If~~ k = 3

$$2^3 - 1 \not\geq 8 + 3$$

~~If~~ k = 4

$$2^4 - 1 \geq 8 + 4 \quad \checkmark \Rightarrow \text{so } k = 4. \text{ (minimum k).}$$

step②

Code word = M + k
= 8 + 4
= 12 bits

	2^3							2^2			2^1	2^0
Bit Positions	12	11	10	9	8	7	6	5	4	3	2	1
Binary Rep.	1100	1011	1010	1001	1000	0111	0110	0101	0100	0011	0010	0001
	D_8	D_7	D_6	D_5	C_8	D_4	D_3	D_2	C_4	D_1	C_2	C_1

step③: Check bits Calculation:

C_1 = XOR all Data bits XXX1
$= D_1 \oplus D_2 \oplus D_4 \oplus D_5 \oplus D_7$

C_2 = XOR all Data bits XX1X
$= D_1 \oplus D_3 \oplus D_4 \oplus D_6 \oplus D_7$

C_4 = XOR all Data bits X1XX
$= D_2 \oplus D_3 \oplus D_8 \oplus D_8$

C_8 = XOR all Data bits 1XXX = $D_5 \oplus D_6 \oplus D_7 \oplus D_8$

$D_8\ D_7 D_6 D_5 D_4 D_3 D_2 D_1$

1 1 0 0 0 0 1 0

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

$C_1 = 0 \oplus 1 \oplus 0 \oplus 0 \oplus 1 = 0$

$C_2 = 0 \oplus 0 \oplus 0 \oplus 0 \oplus 1 = 1$

$C_4 = 1 \oplus 0 \oplus 0 \oplus 1 = 0$

$C_8 = 0 \oplus 0 \oplus 1 \oplus 1 = 0$

$C_8\ C_4\ C_2\ C_1$

0 0 1 0

⇒ Code Word Stored in memory: $D_8\ D_7\ D_6\ D_5\ C_8\ D_4\ D_3\ D_2\ C_4\ D_1\ C_2\ C_1$

1 1 0 0 0 0 0 1 0 0 1 0

C 1 2 H

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* Suppose now that data bit 3 sustains an error and changed from 0 to 1 When check bits re-calculated, we have.

$C_1 = 0 \oplus 1 \oplus 0 \oplus 0 \oplus 1 = 0$

$C_2 = 0 \oplus 1 \oplus 0 \oplus 0 \oplus 1 = 0$

$C_4 = 1 \oplus 1 \oplus 0 \oplus 1 = 1$

$C_8 = 0 \oplus 0 \oplus 1 \oplus 1 = 0$

} new check bits

⇒ When the new check bits compared with stored (old) check bits, the syndrome word is formed:

$C_8\ C_4\ C_2\ C_1$

$\oplus$ 0 0 1 0

0 1 0 0

0 1 1 0

⇒ result is 0110 indicating that bit at position 6 (i.e. D_3) is in error.